THEORY OF TWO-PHASE POROUS COOLING. I

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In conformity with systems of two-phase porous cooling, a statistical model of boiling of a liquid in a porous body consisting of nonintersecting capillaries is set up.

Methods of heat removal from bodies with internal energy sources, based on cooling by a liquid undergoing a phase transformation within a porous wall, have been widely used in recent times. A number of works [1-5] are devoted to the theoretical and experimental investigation of the hydrodynamics and heat exchange in systems of porous two-phase cooling. In the present paper we construct a statistical model of boiling of a liquid inside a porous structure and on the basis of it we formulate the problem of hydrodynamics and heat exchange in systems of two-phase porous cooling. In contrast to [1-4], the approach developed in the present work allows us to take into account the spread of the phase-transition region within the porous body. An account of this fact can turn out to be significant when analyzing instability of systems of two-phase porous cooling, this being connected with a sudden change of the position of the phase-transition region, when the external conditions are altered.

Transfer Equations in a Two-Phase System of Porous Cooling

Let heat sources be distributed inside a porous body (Fig. 1). To remove the heat produced inside and also supplied from outside, a liquid heat carrier undergoing a phase transformation is pumped through it. As we know [1], stability of the system of two-phase porous cooling is achieved by the use of a multilayer porous wall with penetrability increasing in the phase-transformation zone in the direction of motion of the heat carrier. Therefore, in the following it is assumed that penetrability of the wall is a function of the coordinate x. In the general case the phase-transition region is located somewhere in the depth of the body. Before the start of this region, for $x<x_1$, filtration of the liquid heat carrier takes place. At the section $x=x_1$ conditions are created for the formation of vapor bubbles in individual capillaries and for their subsequent growth. Beginning with this location and at the distance $\Delta = x_2 - x_1$, a phase transition takes place. For $x>x_2$ a single-phase flow of the already gaseous heat carrier takes place.

If we can introduce a single temperature for the porous matrix, liquid, and vapor located in the pores (this is possible if their temperatures are close to one another), then the densities of the flow of mass, momentum, and energy in the direction of the x axis can be represented in the form

$$\tau_m = \rho_v v_v + \rho_l v_l, \tag{1}$$

$$\Sigma = p + \sigma_x, \tag{2}$$

$$q = \rho_v v_v h_v + \rho_l v_l h_l - \lambda' \frac{dT}{dx}, \tag{3}$$

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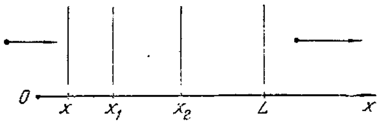

Fig. 1

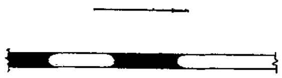

Fig. 2

Fig. 1. Filtration scheme of a heat carrier in two-phase porous cooling system.

Fig. 2. Flow of a heat carrier in a uniaxial capillary.

where σ_x is the flow of momentum in the direction of the x axis transferred as a result of the motion of the heat carrier. With the expressions (1)-(3) taken into account, the corresponding stationary transfer equations are written in the form

$$\frac{d}{dx}(\rho_v v_v + \rho_l v_l) = 0, \tag{4}$$

$$\frac{dp}{dx} = -\frac{d\sigma_x}{dx} + \tau_\omega, \tag{5}$$

$$\frac{d}{dx}[\rho_v v_v h_v + \rho_l v_l h_l] - \frac{d}{dx}\left(\lambda' \frac{dT}{dx}\right) = r(x), \tag{6}$$

where τ_ω is the friction force acting on the heat carrier per unit volume from the side of the porous matrix; r(x) is the density of internal heat sources. Equations (4)-(6) do not define uniquely all the quantities contained by them. To close this system of equations we have to use model representations of the character of boiling of the heat carrier in the porous body.

Statistical Model of Boiling Process of Liquid in a Porous Body

Let a porous body consist of a system of identical cylindrical capillaries arranged parallel in the direction of the x axis. In experiments concerned with the study of boiling of a liquid in individual capillaries [6], a periodic emergence of vapor bubbles covering the entire section of capillaries is discovered. The bubbles are formed on the walls of capillaries at centers of vapor information that exist there (Fig. 2). They rapidly grow as a result of evaporation from the menisci of liquid located at the ends of the vapor plug. This plug moves along the capillary together with the stream of liquid, but the column of liquid in front of the vapor plug moves considerably faster as a result of the growth of the plug. According to experiments, a simultaneous occurrence of several bubbles in the same capillary is unlikely. After evaporation of the column of liquid in front of the vapor plug activation of a new bubble takes place. The probability of bubble activation depends on the degree of superheating of the liquid and numerous other factors. The state of the surface of the capillary, the degree of purity of the liquid, etc., are among such factors. For the formation of bubbles, the liquid, first of all, must be superheated, i.e., must have a temperature higher than the saturation temperature at the given pressure $T_0 = T_{sat}(p)$. Let the liquid be superheated up to the temperature T. Then spherical equilibrant bubbles of radius

$$r = \frac{2\sigma}{T - T_0} \cdot \frac{dT_{sat}(p)}{dp} \tag{7}$$

can exist in it. Bubbles of smaller radius will decrease, while bubbles of larger radius will grow. In other words, centers of vapor formation on the surface of a capillary of radius $R > r$ can be activated, while those with a dimension $R < r$ cannot. Let centers of vapor formation per unit surface of the capillary be distributed with respect to the size with the distribution density n(r). Then we can determine the probability of activation of any center of vapor formation with a dimension greater than r:

$$\omega(r) = \int_r^\infty n(u)\,du \Big/ \int_0^\infty n(u)\,du, \tag{8}$$

where r is given by the expression (7). The quantity $\omega(r)$ depends on the superheating $T - T_0$ and through it also on the coordinate x. Therefore, in the following this quantity will be denoted by $\omega(x)$.

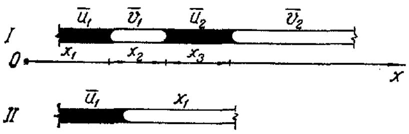

Fig. 3

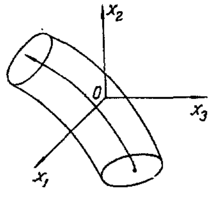

Fig. 4

Fig. 3. Flow of the heat carrier in capillaries of the first and second types.

Fig. 4. Evolution of capillaries of the first type in the phase space.

Since in the same capillary simultaneously no more than a single bubble can be activated, all capillaries passing through a unit section of the porous body are divided into two groups (Fig. 3): those with a vapor plug and those without it. The capillaries of the first group can be characterized by a triplet of numbers: x_1 – the coordinate of the first meniscus; x_2 – the length of the vapor plug; and x_3 – the length of the liquid column behind the plug. The capillaries of the second group are adequately characterized by a single number – the coordinate x_1 of the meniscus. In the space x_1, x_2, x_3 we introduce the density of capillaries N, equal to the number of capillaries of the first group passing through a unit section of the specimen and having the characteristics x_1, x_2, x_3. An analogous characteristic is introduced for capillaries of the second group N_1. These quantities depend not only on coordinates, but can also depend on time.

Capillaries of one group can be transformed into capillaries of the second group as a result of evaporation of the column of liquid in capillaries of type 1 or as a result of emergence of a bubble of vapor in capillaries of type 2. Capillaries of type 1 appear from capillaries of type 2 at the instant of occurrence of the vapor plug. For capillaries in which this occurs $x_2 = 0$. Subsequently the parameters x_1, x_2 grow, while x_3 diminishes. When x_3 becomes equal to zero, the capillary of type 1 is transformed into a capillary of type 2 (Fig. 4). With the growth of x_i the functions N and N_1 must rapidly decrease; therefore, we can assume that the region of variation of the arguments is $-\infty < x_1 < \infty$; $x_2 > 0$; $x_3 > 0$.

The flow in the phase space x_1, x_2, x_3 is defined as $\Phi_i = v_i N$, where v_i is the rate of variation of the coordinate x_i. Then for N the balance equation

$$\frac{\partial N}{\partial t} + \sum_{(i)} \frac{\partial}{\partial x_i} (v_i N) = Q \tag{9}$$

holds, where Q is a quantity equal to the number of capillaries of the first type with the parameters x_1, x_2, x_3 being formed per unit time in unit section of the specimen. The quantity $Q(x_1, x_2, x_3)$ must be proportional to the number of capillaries of the second type with a meniscus at the point $x_1 + x_3$. Since capillaries of the first type arise from $x_2 = 0$, we can represent Q in the form

$$Q = k\omega(x_1)\, N_1(x_1 + x_3)\, \delta(x_2), \tag{10}$$

where in the coefficient of proportionality we have isolated a term depending on the superheat of liquid $\omega(x_1)$. Equation (9) is now written in the form

$$\frac{\partial N}{\partial t} + \sum_{(i)} \frac{\partial}{\partial x_i} (v_i N) = k\omega(x_1)\, N_1(x_1 + x_3)\, \delta(x_2). \tag{11}$$

Since we are interested in the stationary case, for the determination of N it is sufficient to have the equations

$$\sum_{(i)} \frac{\partial}{\partial x_i} (v_i N) = k\omega(x_1)\, N_1(x_1 + x_3)\, \delta(x_2) \tag{12}$$

with the condition

$$N|_{x_2=-0} = 0. \tag{13}$$

The condition (13) reflects the fact that capillaries of type 1 arise in the plane x_1, x_3 of the phase space x_1, x_2, x_3.

Capillaries of type 2 arise from capillaries of type 1 at the instant the liquid column evaporates. They vanish when they are transformed into capillaries of the first type. The balance equation for N_1 has the form

$$\frac{\partial N_1}{\partial t} + \frac{\partial}{\partial x_1} (N_1 v_1') = R(x_1) - \bar{R}(x_1), \tag{14}$$

where $R(x_1)$ is a source reflecting the transition of capillaries of type 1 into capillaries of type 2; $\bar{R}(x_1)$ is a source characterizing the inverse transition; v'_1 is the rate of motion of the meniscus in a capillary of the second type. The number of capillaries of the second type with a meniscus at the point x_1, being transformed into capillaries of type 1 with any lengths of the liquid column and a vapor plug, is given by the expression

$$\bar{R}(x_1) = \int_0^\infty k\omega(x_1 - x_3) N_1(x_1)\, dx_3. \tag{15}$$

The number of capillaries of the second type, arising from capillaries of the first type with any length of the plug, is written in the form

$$R(x_1) = \int_0^\infty v_3 N|_{x_3=0}\, dx_2. \tag{16}$$

With (15), (16) taken into account, Eq. (14) in the stationary case assumes the form

$$\frac{d}{dx_1}(v_1' N_1) = \int_0^\infty v_3 N|_{x_3=0}\, dx_2 - \int_0^\infty k\omega(x_1 - x_3) N_1(x_1)\, dx_3. \tag{17}$$

Equations (12) and (17) with the condition (13) determine the functions N and N_1 under the additional condition

$$\int_0^\infty \int_0^\infty \int_{-\infty}^\infty N dx_1 dx_2 dx_3 + \int_{-\infty}^\infty N_1(x_1)\, dx_1 = N_0, \tag{18}$$

where N_0 is the overall number of capillaries passing through a unit section of the specimen.

Transfer Equations on the Basis of a Statistical Model of the Boiling Process in a Capillary-Porous Body

On the basis of the statistical model of boiling in a porous body we determine all quantities entering into the transfer equations (4)-(6). Let the rate of motion of the liquid at the entry into a capillary be $\bar{u}_1$. For simplification we shall assume that it is the same for capillaries of the first and second types and that $v_1 = v_1'$. The rate of motion of vapor in the vapor plug $\bar{v}_1$ is greater than $\bar{u}_1$. In addition, $\bar{u}_2 > \bar{v}_1$, $\bar{v}_2 > \bar{u}_2$. Having written for each of the menisci the condition of continuity of the flow of material, which, for example, for the first of them has the form

$$\rho_l (\bar{u}_1 - v_1) = \rho_V (\bar{v}_1 - v_1) = \tau_1, \tag{19}$$

where τ_1 is the rate of evaporation, representable, according to [7], in the form

$$\tau = \frac{\varkappa\alpha\sqrt{\mu}}{\sqrt{2\pi R^* T}} [p(T) - p], \tag{20}$$

by means of several algebraic transformations, with the relation $\rho_V \ll \rho_l$ taken into account, we find that

$$\bar{v}_1 = \bar{u}_1 + \frac{\tau_1}{\rho_{V_1}}, \tag{21}$$

$$\bar{u}_2 = \bar{u}_1 + \frac{\tau_1}{\rho_{V_1}} + \frac{\tau_2}{\rho_{V_2}}, \tag{22}$$

$$\bar{v}_2 = \bar{u}_1 + \frac{\tau_1}{\rho_{V_1}} + \frac{\tau_2}{\rho_{V_2}} + \frac{\tau_3}{\rho_{V_3}}, \tag{23}$$

$$v_1 = v_1' = \bar{u}_1; \quad v_2 = \frac{\tau_1}{\rho_{V_1}} + \frac{\tau_2}{\rho_{V_2}}; \quad v_3 = -\frac{\tau_2 + \tau_3}{\rho_l}. \tag{24}$$

We introduce the quantities n_1, n_2, n_3, n_4, equal to the mean number of capillaries per given section of the specimen x, respectively, with an initial part filled with liquid, a vapor plug, a liquid column behind the vapor plug, and the final part filled with vapor. It is not difficult to see that these quantities are expressed as follows in terms of the functions $N(x_1, x_2, x_3)$ and $N_1(x_1)$:

$$n_1(x) = \int\limits_x^\infty N_1(x_1)\,dx_1 + \int\limits_0^\infty \int\limits_0^\infty \int\limits_x^\infty N(x_1,\ x_2,\ x_3)\,dx_1dx_2dx_3, \tag{25}$$

$$n_2(x) = \int\limits_{-\infty}^x N_1(x_1)\,dx_1 + \int\limits_0^\infty \int\limits_x^\infty \int\limits_{-\infty}^x N(y_1,\ y_2-y_1,\ y_3)\,dy_1dy_2dy_3, \tag{26}$$

$$n_3(x) = \int\limits_x^\infty \int\limits_{-\infty}^x \int\limits_{y_1}^x N(y_1,\ y_2-y_1,\ y_3-y_2)\,dy_2dy_1dy_3, \tag{27}$$

$$n_4(x) = \int\limits_{-\infty}^x \int\limits_{y_1}^x \int\limits_{y_2}^x N(y_1,\ y_2-y_1,\ y_3-y_2)\,dy_3dy_2dy_1. \tag{28}$$

By means of the quantities thus introduced we can express the density of the flow of vapor, the density of the flow of liquid, and the total density of the mass flow of the heat carrier as follows:

$$\rho_v v_v = s\,(n_2\rho_v\bar{v}_1 + n_4\rho_v\bar{v}_2), \tag{29}$$

$$\rho_l v_l = s\,(n_1\rho_l\bar{u}_1 + n_3\rho_l\bar{u}_2), \tag{30}$$

$$\tau_m = s\,(n_1\rho_l\bar{u}_1 + n_2\rho_v\bar{v}_1 + n_3\rho_l\bar{u}_2 + n_4\rho_v\bar{v}_2). \tag{31}$$

The density of the impulse flow connected with the convective motion of the heat carrier is represented in the form

$$\sigma_x = s\,(n_1\rho_l\bar{u}_1^2 + n_2\rho_v\bar{v}_1^2 + n_3\rho_l\bar{u}_2^2 + n_4\rho_v\bar{v}_2^2). \tag{32}$$

If for λ' the additive law of compounding thermal conductivities is true, then we can write

$$\lambda' = \lambda_v s\,(n_2 + n_4) + \lambda_l s\,(n_1 + n_3) + \lambda_{\text{p.m}}\Big(1 - \sum_{i=1}^{4} n_i\Big). \tag{33}$$

It remains for us to determine the friction force acting on the heat carrier in a unit volume of the porous body. For this we introduce the quantities τ_{ω_1}, τ_{ω_2}, τ_{ω_3}, τ_{ω_4} equal to the friction force acting from the side of unit surface of the capillary on the heat carrier in the given section x, dependent on which side of the capillary falls onto the given section. Then it is obvious that

$$\tau_\omega = a\,(n_1\tau_{\omega_1} + n_2\tau_{\omega_2} + n_3\tau_{\omega_3} + n_4\tau_{\omega_4}). \tag{34}$$

The quantity $\tau_{\omega i}$ is expressed in terms of the coefficient of friction [8]

$$\xi = \frac{8\tau_\omega}{\rho v^2}, \tag{35}$$

being a function of the number Re of the flow of liquid moving along the capillary, $\xi = \xi(\text{Re})$. In the case of laminar flow this function has the form

$$\xi = \frac{32}{\text{Re}}, \quad \text{Re} = \frac{uR}{\nu}.$$

From (35) we obtain

$$\tau_{\omega_1} = \frac{\rho_l \bar{u}_1^2}{8}\,\xi(\text{Re}_1), \quad \text{Re}_1 = \frac{\bar{u}_1 R}{\nu_l}.$$

Analogous expressions can be written for τ_{ω_2}, τ_{ω_3}, τ_{ω_4}.

Thus, equations of motion are closed in the framework of the model of boiling in a capillary-porous body formulated above. Concluding, we note that the theory developed here is valid also for a porous body consisting of identical capillaries of variable section, simulating a stable system of two-phase porous cooling with penetrability increasing in the direction of motion of the heat carrier.

NOTATION

ρ, density; v, velocity; p, pressure; T, temperature; τ, density of the mass flow; Σ, density of impulse flow; q, density of heat flow; h, enthalpy; λ', effective coefficient of thermal conductivity; σ, surface-tension coefficient; v_1, v_2, v_3, rates of change of the coordinates x_1, x_2, x_3, respectively; $\bar{u}_1$, $\bar{u}_2$, average velocities of liquid in front of the first meniscus and column of liquid, respectively; $\bar{v}_1$, $\bar{v}_2$, velocities of vapor in the vapor plug and at the exit from the capillary; α, condensation coefficient; μ, molecular weight; R*, universal gas constant; P(T), saturated vapor pressure at the temperature T; λ_v, λ_l, $\lambda_{p.m}$, coefficients of thermal conductivity of vapor, liquid, and material of the porous matrix.

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COMBINED MEASUREMENT OF THERMAL PROPERTIES OF FLUIDS

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A dilatometric method for simultaneous determination of the heat capacity and the coefficient of thermal expansion of fluids is described.

Rapid dilatometric methods for the determination of the thermal diffusivity of materials [1], which have demonstrated undisputed advantages in the study of heat-transfer parameters [2], can be used as a means for combined measurement of the properties of thermal expansion and heat capacity in fluids [3].

Among the principal proposals for such an expansion in the area of application of dilatometry (while preserving such important qualities as the nondestructive and highly accurate nature of the method, which is not based on measurement of temperatures and thermal fluxes in test objects), one should consider the comparative version involving thermal change in the volume of two fluids – a standard (with known values for the volumetric heat capacity c_0 and for the coefficient of thermal expansion β_0) and a test fluid (the thermophysical characteristics c_T and β_T of which are subject to determination) – under conditions where the variation of heat content in each of them occurs only because of heat exchange through a boundary separating the fluids (a thin nondeformable shell).

As a specific model for the realization of the method, it is convenient to select a system of two "imbedded" thin-walled metal vessels made of a material with a negligibly small coefficient of thermal expansion in comparison with the same parameter for the fluids. The fluid with standard properties fills the outer vessel 1 (Fig. 1) in such a way that the inner vessel 3 is completely immersed in the standard fluid, which is in

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